# American University of Sharjah

## Department of Mathematics and Statistics

## MTH 203, SAMPLE FINAL EXAM - 2009-2012

### MULTIPLE CHOICE SECTION

(You must circle ALL correct statements. No justification is required.

1. (5pts) If $\mathbf{v}$ and $\mathbf{w}$ are vectors then $\mathbf{w} \cdot (\mathbf{v} \times \mathbf{v})$ equals
   (a) 1
   (b) 0
   (c) $||\mathbf{v}||||\mathbf{w}||^2$
   (d) not a defined expression
   (e) not enough information

2. (5pts) If $\mathbf{v}$ and $\mathbf{w}$ are vectors then $\mathbf{v} \times (\mathbf{w} \cdot \mathbf{w})$ equals
   (a) 1
   (b) 0
   (c) $||\mathbf{v}||||\mathbf{w}||^2$
   (d) not a defined expression
   (e) not enough information

3. (5pts) The equation of the plane passing through the points $(2, 2, 2), (2, 1, 1)$ and $(2, 2, 1)$ is (*Hint: You can anser this with no calculations*).
   (a) $(x - 2) + (y - 2) + (z - 2) = 0$
   (b) $x + y = 2$
   (c) $x = 2$
   (d) $(y - 1) - (z - 1) = 0$
   (e) none of the above

4. (5pts) The limit $\lim_{(x,y)\to(0,0)} \dfrac{2x^2y}{x^3 + xy^2}$ equals
   (a) 0
   (b) 1
   (c) $-1$
   (d) $\infty$
   (e) $-\infty$
   (f) None of the above.

5. (5pts) The level curve of $f(x, y) = e^{x+y}$ passing though the point $(0, e)$ is

(a) a parabola
(b) an exponential curve
(c) a line
(d) a log curve
(e) the shape of your professors head

6. (5pts) Which of the following is a normal vector to the curve $y = x^2$ at the point $(2, 4)$
   (a) $\langle 4, 1\rangle$
   (b) $\langle 8, -2\rangle$
   (c) $\langle 1, 4\rangle$
   (d) $\langle 1, 1\rangle$
   (e) $\langle -2, 1\rangle$

7. (5pts) If $f(x, y) = x^2 - y^2 - x^4$ then $f(x, y)$ has which types of critical points.
   (a) one saddle point
   (b) one saddle point and one local maximum
   (c) one saddle point and two local maximum
   (d) one saddle point and one local minimum
   (e) one saddle point and two local minimum
   (f) none of the above.

8. (5pts) The integral $\int_0^1 \int_0^{x^2+1} f(x, y)\, dydx$ equals
   (a) $\int_0^2 \int_{\sqrt{y-1}}^1 f(x, y)\, dxdy$
   (b) $\int_1^2 \int_{\sqrt{y-1}}^1 f(x, y)\, dxdy$
   (c) $\int_0^2 \int_0^{\sqrt{y-1}} f(x, y)\, dxdy$
   (d) $\int_1^2 \int_0^{\sqrt{y-1}} f(x, y)\, dxdy$
   (e) $\int_0^{x^2+1} \int_0^1 f(x, y)\, dxdy$
   (f) None of the above

9. (5pts)Let $f(x, y) = xe^{xy^2} + 2$. The linear approximation of $f(x, y)$ at the point $(0, 1)$ is
   (a) $L(x, y) = 2 + (e^{xy^2} + xy^2e^{xy^2})(x - 1) + 2x^2ye^{xy^2}y$
   (b) $L(x, y) = (e^{xy^2} + xy^2e^{xy^2})\Delta x + 2x^2ye^{xy^2}\Delta y$
   (c) $L(x, y) = x$
   (d) $L(x, y) = 2 + x$

(e) None of the above

10. The rate of change of $f(x,y) = 3x + y^2$ at the point $(-4,2)$ in the direction $\langle 3,4\rangle$ is

(a) 25
(b) 5
(c) $\langle 3,4\rangle$
(d) $7\langle 3,4\rangle$
(e) none of the above

1. Find the directional derivative of $f(x,y) = e^{x\sin y}$ at $P(1,0)$ in the direction to $Q(2,4)$.

(a) $\frac{4}{5}$ (b) $\frac{-3}{5}$ (c) $-2$ (d) 0 (e) None of these

2. Let $f(x,y) = 8x^3 + 6xy + y^3$. Which of the following statements is true about the critical points of $f$?

(a) (0.0) is a saddle pt. and $(\frac{-1}{2}, -1)$ is a local max
(b) (0.0) is a saddle pt. and $(\frac{-1}{2}, -1)$ is a local min
(c) (0.0) is a local max and $(\frac{-1}{2}, -1)$ is a local min
(d) (0.0) is a local min and $(\frac{-1}{2}, -1)$ is a local max
(e) None of these

3. Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^4y^2}{x^4+3y^4}$

(a) 2 (b) 0 (c) $\frac{2}{3}$ (d) $\frac{1}{2}$ (e) None of these

4. If $yz^3 + ze^{xy} = \cos x$, find $\dfrac{\partial z}{\partial x}$

(a) $\dfrac{\sin x}{3x^2 - e^{xy}}$ (b) $\dfrac{y\cos x - ze^{xy}}{xz^2 - ye^{xy}}$ (c) $\dfrac{\cos x - e^{xy}}{z^2 - ye^{xy}}$ (d) $\dfrac{-\sin x - yze^{xy}}{3yz^2 + e^{xy}}$ (e) None of these

5. What is the direction in which the function $f(x,y) = yx^2 - \frac{x}{y^2}$ increases most rapidly at the point $(-2,1)$?

(a) $\langle -1,0\rangle$ (b) $\langle 3,2\rangle$ (c) $\langle 0,0\rangle$ (d) $\langle 3,1\rangle$ (e) None of these

6. Find the equation of the plane containing the line $r(t) = (1+t, 3-2t, -2+6t)$ and the point $P(-3,1,6)$

(a) $x - 8y + 3z = -29$ (b) $x - 8y = 25$ (c) $-2x + 16y + 5z = 33$ (d) $2x + 16y + 5z = 40$ (e) N

7. Compute the integral $\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \sqrt{x^2+y^2}\,dy\,dx$

(a) $\dfrac{1}{2}\pi$ (b) $\dfrac{9}{2}\pi$ (c) $9\pi$ (d) $\dfrac{81}{8}\pi$ (e) None of these

8. A wagon is pulled a distance of 20 m along a horizontal path by aconstant force of 35 N. The handle of the wagon is held at an angle of 30 degrees above the horizontal. How much work is done?

(a) 350 (b) $350\sqrt{3}$ (c) 700 (d) 45 (e) None of these

9. Which of the following statement about the planes $x + y - z = 2$ and $2x - y + 3z = 1$ is true?

   (a) parallel (b) intersect at the line $x = \frac{7}{5} - 2t,\ y = 5t,\ z = \frac{-3}{5} + 3t$ (c) skewed

   (d) intersect at the line $x = \frac{2}{5} - t,\ y = 1 + 5t,\ z = -\frac{1}{5}t$ (e) None of these

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**Written Part**

(You must show your work and justify your answers.)

1. (10pts) Consider the force field
$$\mathbf{F} = \langle 2xy - 1, x^2 \rangle$$
and let the curve $C$ be the graph of the function $y = \sin(x)$ joining $(0, 0)$ to $(\pi, 0)$.

   Compute the work done by $F$ along $C$

   *Hint: Use the fundamental theorem of line integrals.*

2. (10pts) Let $\mathbf{F} = \langle xy, y^2 \rangle$.

   Let $C_1$ be the counter-clockwise circle of radius $a$ centered at $(0, 0)$.

   (a) Directly compute $\int_C \mathbf{F} \cdot d\mathbf{r}$.

   (b) Is $\mathbf{F}$ conservative? Justify your answer.

   (c) Find a **closed curve** $C_2$ so that $\int_{C_2} \mathbf{F} \cdot d\mathbf{r} \neq 0$. You may describe $C$ by a sketch or equations but you must justify your answer. *Hint: Use Green's theorem.*

3. (10 Points) Let $E$ be the solid bounded by the surfaces $z = 4 - x^2 - y^2$ and the $xy$-plane. Let $\mathbf{F} = \langle -y, x, 1 \rangle$.

   (a) The surface bounding $E$ is made of two parts. Compute the flux over the lower part.

   (b) Compute the flux over the upper part (that is, the part with $z > 0$).

   (c) Compute $\iiint_E \operatorname{div} \mathbf{F}\ dV$. Did you get the answer you expected? Explain using the divergence theorem.

   (d) (10 pts) Let $\mathbf{F} = \langle x + z, y + \sin(z), e^{x^2} z \rangle$.

   Let $S$ be the top half of the ellipsoid $x^2 + y^2 + 4z^2 = 1$ with $z \geq 0$. Compute the flux
$$\iint_S \operatorname{curl} \mathbf{F} \cdot \mathbf{n}\, dS$$
   upward through the surface $S$.

   *Hint: The direct evaluation of this integral is impossible. So please don't try!*

1. (6.5 Points) **Set up** the integral needed to compute the volume of the solid under the paraboloid $z = x^2 + y^2$ and above the region bounded by $y = x^2$ and $y = x$.

2. (8 Points) **Find** the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 9$, above the $xy$-plane, and below the cone $z = \sqrt{x^2 + y^2}$.

3. Let $\mathbf{F}(x, y, z) = \langle e^y,\ xe^y + e^z,\ ye^z \rangle$ .

   (a) (4 Points) Why $\mathbf{F}$ is a conservative vector field?

   (b) (5 Points) Find the potential function $f(x, y)$.

   (c) (5 Points) Use the *Fundamental Theorem of Line Integral* **to compute** the work done by the force $\mathbf{F}$ to move an object along a smooth curve $C$ joining $(0, 2, 0)$ to $(4, 0, 3)$.

4. (8 Points) **Evaluate** the line integral $\int_C x^3 z \, ds$, where $C$ is the curve given by $\mathbf{r}(t) = \langle 2 \sin t, t, 2 \cos t, \rangle$ , $0 \leq t \leq \frac{\pi}{2}$

5. (8 Points) Use the *Divergence Theorem* **to set a formula** that computes the outward flux of the vector field $\mathbf{F}(x, y, z) = \langle x^2, xy, x^3y^3 \rangle$ through the surface of the tetrahedron $x + y + z = 1$ and the planes $x = 0$, $y = 0$, and $z = 0$.

6. (10 Points) Use *Green's Theorem* to **evaluate** $\int_C xy \, dx + x^2y^3 dy$ where $C$ is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 2)$.